

Investigating upper limb perceptual asymmetries for unconstrained active exploration of stiffness cues

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Abstract—While many factors shape the preferential use of our hands, there is growing evidence that each hand may also perceive the environment differently. Currently, this has been demonstrated for proprioceptive and cutaneous cues, but our understanding of perceptual asymmetries for kinesthetic cues like stiffness is limited. In this manuscript, we measured JNDs of $N=14$ participants in an active stiffness discrimination task using their left and right hand. We found significant perceptual asymmetries between the two hands with left hand exploration leading to lower JNDs. Further investigation is needed, however, to understand the potential role of handedness in the observed perceptual asymmetries.

I. INTRODUCTION

Humans regularly explore their environments with their hands in a variety of ways. Active exploration strategies can be used to perceive different objects [1], [2], determine their physical properties like shape, size and texture [3], [4], and estimate their mechanical properties like stiffness, damping, and mass [5], [6]. In unimanual exploration, either of the two hands can be used to evaluate the same parameters. Even seemingly simple day to day tasks like comparing ripeness of two fruits can involve comparing two independent unimanual percepts from each hand. The choice of exploration strategy depends on multiple factors including user preference, task-specific conditions, and convenience.

We know that haptic perceptual asymmetries exist in the upper limb and existing literature on unimanual perception focuses primarily on comparisons of percepts for proprioceptive and cutaneous cues [1], [2], [7], [8], from passive stimulation. Several studies have reported significant differences in perception between the two hands for these cues [7], [8]. For example, there is empirical evidence to suggest that our perception of curvature and length may depend on the hand used for exploration [1], [2]. In addition, perceptual asymmetries with a consistent non-dominant hand advantage have also been reported in studies involving movement tasks without visual feedback [9]–[11]. Likewise, Goble et al. have shown perceptual asymmetries for proprioceptive cues [12], [13]. It is unclear, however, if these perceptual asymmetries between the hands also exist for actively explored kinesthetic haptic cues.

In this manuscript, we begin to fill this knowledge gap by investigating stiffness perception thresholds for active

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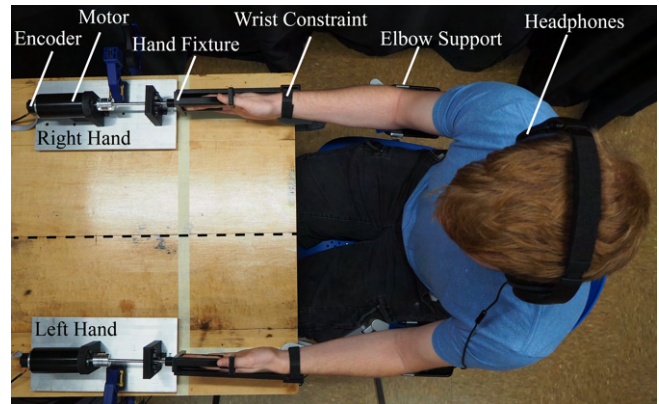


Fig. 1. Experimental apparatus set up for with identical 1-DoF rotational kinesthetic haptic interfaces for both hands. Only one hand was used at a time during the experiment, based on the exploration condition. A pair of headphones were used for audio cues.

unimanual exploration. Using two identical 1-DoF rotational kinesthetic haptic devices [14] and the psychophysical method of constant stimuli [15], [16], we evaluated participants' ability to discriminate virtual torsion springs under the following two exploration conditions: 1) unimanual exploration via rotation of the left hand and 2) unimanual exploration via rotation of the right hand. Active exploration with roving displacements (controlled by the participant) was chosen here for its practical significance in our daily exploration of our environment, as well as to limit the adverse effects of fixed displacements on performance in stiffness discrimination tasks [17]. In this way, the exploration strategy aligns well with the manner in which we explore real torsion springs in our environment. We compare discrimination thresholds for the two conditions and compare our results to those obtained in the literature for other haptic cues.

We hypothesize that stiffness perception thresholds for the two unimanual conditions will be significantly different based on the similar perceptual asymmetries observed for proprioceptive and cutaneous cues in literature [8], [12], [13], [18]. In what follows, we present our experimental protocol and findings, followed by a discussion of our results in the context of current literature on haptic perception under unimanual exploration. We also discuss their potential applications in haptic feedback mechanisms and their potential implications in psychophysical assessment of haptic perception of the upper limbs.

II. METHODS

A. Participants

We recruited $n=14$ individuals (8 male, 6 female, age = 23 ± 4 years) to perform a psychophysical task of stiffness discrimination between virtual torsional springs. All participants provided written informed consent according to a protocol approved by the Johns Hopkins School of Medicine Institutional Review Board (Study# IRB00148746). Participants were compensated at a rate of \$10/hour. All participants were right-hand dominant with no history of upper limb impairments.

B. Apparatus

We used a custom direct drive 1-DoF rotary kinesthetic haptic device (see Fig. 1) for this study. This device has been used in our prior work [14] and features a Maxon RE50 (200 Watt) motor equipped with a 3-channel Maxon HEDL Encoder (500 CPT) encoder, and was driven by a Quanser AMPAQ-L4 Linear current amplifier. The apparatus is capable of generating a peak torque of 467 mNm. Data acquisition and control was provided through a Quanser QPIDE PCI data acquisition board with a MATLAB/Simulink and Quarc real-time software interface run at a frequency of 1KHz. Two pairs of Bose headphones were used to provide identical audio cues to the experimenter and the participant.

A custom 3D printed hand fixture, attached via a rotary shaft, served as the primary mode of interaction for the participant. The fixture enabled an alternating finger pattern to maintain a uniform grip throughout the experiment for each participant, and across all participants. A Velcro strap was used to fix the participant's forearm to the hand fixture to limit wrist flexion/extension and radial/ulnar deviation. The elbow was placed on a height-adjustable support to align the forearm's rotational axis with the device's rotational axis. In this way, the device and the support enable exploration that is 1-DoF in both, task-space and joint-space. The device was programmed to produce a torque that linearly increases with the participant's rotational displacement, rendering the sensation of exploring a torsional spring, as discussed below.

C. Experiment Design

The method of constant Stimuli was used to evaluate participants' ability to differentiate between torsion springs in a 2 Alternate-2 Interval Forced Choice (2A-2IFC) experiment. An asymmetric design with catch trials was used to determine stiffness discrimination thresholds. In the experiment, participants were asked to identify which of the two sequentially presented springs in a given pair was stiffer.

The virtual torsion spring was grounded with respect to the participant's hand. Pronation of the hand resulted in compression of the virtual torsion spring. Torsion springs and hand rotation were chosen specifically to limit haptic exploration to a single degree of freedom, reducing any potential proprioceptive confounds. Participants actively explored the springs, which were rendered according to the following Hooke's Law formulation:

$$\tau = c_{test} \cdot k_{ref} \cdot \Delta\theta \quad (1)$$

where τ is the output torque of the motor, $\Delta\theta$ is the angular displacement of the participant's hand from the neutral position, k_{ref} is the spring constant of the reference virtual spring, and c_{test} is a scaling factor used to render different virtual test springs, as required by the psychophysical paradigm.

For the comparison spring pairs, an asymmetric designed was used with a single reference spring of spring constant of 2 mNm/deg. Five test springs were selected at equally spaced intervals of 5% with stiffness values ranging from 105% to 125% of the reference (2.1 mNm/deg to 2.5 mNm/deg). Each of the five spring pairs was presented ten times in random order for a total of 50 trials. The order in which reference and test springs were presented was balanced across all trials and for all test springs. Additionally, five catch trials were randomly introduced in each block. Each catch trial required discriminating between two randomly presented springs of 1.45 mNm/deg and 2.45 mNm/deg stiffness, respectively. The experiment was terminated if participants incorrectly identified the 1.45 mNm/deg spring as stiffer than the 2.45 mNm/deg spring more than once, as this error likely suggested a significant response bias, lack of concentration, or lack of understanding of the experiment. The reference spring was not used in the interleaved catch trials to reduce the participants' ability to discern that the same reference spring was used for all other experimental trials.

The number of test stimuli (5) and number of trials per stimuli (10) are consistent with the recommendations made in [16], [19] for 2AFC psychophysical experiments and prior haptics literature [20], [21]. The use of catch trials is also consistent with the choice of asymmetric design for the method of constant stimuli. The stiffness values were determined from preliminary empirical testing using a simple Yes/No detection experiment with three volunteers using their right hand on the same setup. The resulting values were chosen to ensure that the stimuli were considerably higher than an average participants' absolute detection threshold, but not too high to reduce the potential confounds from muscle fatigue over the course of the experiment.

D. Procedure

Once the participants provided the informed consent to enroll in the study, they completed a demographic questionnaire and the Edinburgh handedness survey to determine their laterality index (LI) (See Table I). An LI in the range (30 to 100) suggests right hand dominance and an LI in the range (-30 to -100) suggests left hand dominance. Participants were instructed to adjust their seat height and posture to maintain elbow flexion at 90 degrees with limited and symmetric abduction for both upper arms. The participants then inserted their hands in the fixture and the experimenter secured it in place with a Velcro strap. The elbow support was adjusted such that it only made contact at the Olecranon (tip of the elbow). Significant attention was given to the posture of the

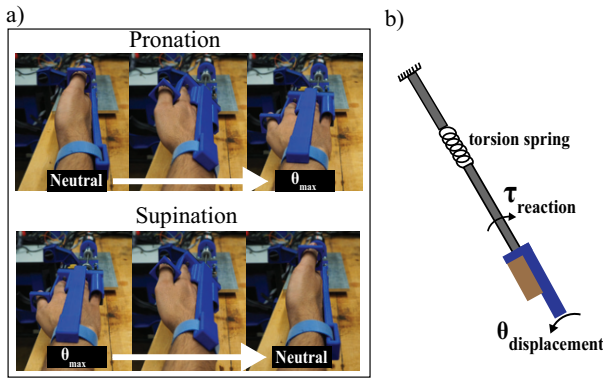


Fig. 2. a) Standard right hand unimanual exploration strategy, including pronation from neutral position to maximum angular displacement (θ_{max}), determined freely by the participant for each exploration, and supination back to neutral position. b) Representation of a torsion spring providing reactionary torque to a right hand performing pronation.

participant to ensure that the same posture was maintained for each hand. Participants were asked to rest the arm that was not in use on their lap. Once the participants were seated, they were instructed to look at a nondescript white wall in front of them with explicit additional instructions to not look at the experimental setup during the experiment in order to avoid any visual confounds [14]. Participant compliance was monitored by the experimenter.

To explore the springs, participants were instructed to start from the neutral position, pronate their hand, and then supinate back to the neutral position as shown in Fig. 2a. Participants were allowed to pronate to an angular displacement of their choice, at their desired velocity for each trial, while adhering to the following instructions: 1) to complete the exploration in one smooth motion, 2) to avoid lifting their elbow, and 3) to avoid going beyond the neutral position during supination, to maintain a consistent starting point for the next exploration. Participants were asked to feel each torsion spring at least twice, with no limit on maximum number of explorations. Audio cues from the headphones informed the participants if they were exploring the first or the second spring of the pair, and gave them “start” and “stop” instructions. Participants were given four seconds to explore each spring, but were not informed of the exact time between the two audio commands to discourage them from consciously tracking the time. Trials were repeated without changing the spring pair if participants failed to complete their exploration within the time constraints, however, the participant was not informed that they were exploring the same pair of springs again.

The order of presentation for the Left and Right exploration blocks was randomized. Each block started with a training period where the participant was allowed to gain familiarity with the setup on a randomly selected spring. Once the participant was able to follow the required exploration strategy with the audio cues in the four second window, they performed three training trials on three pairs of randomly selected springs. The experiment phase of the

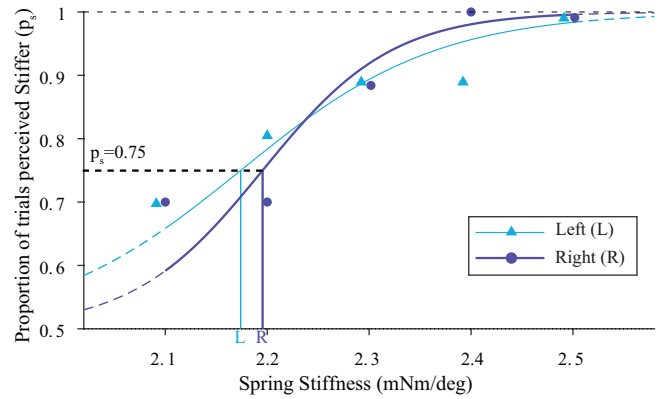


Fig. 3. Psychometric curves fit to a select participant's responses using a logistic probability distribution. Points L and R represent the stiffness values corresponding to the 75% proportion correct response rate for the Left and Right exploration conditions, respectively.

block was only started after the participant could follow the experimental protocol for three consecutive training trials. During the experiment, participants were not provided any feedback on the correctness of their response for any of the trials. Each block had a two-minute mid-session break, and a five-minute break was provided between the blocks.

E. Metrics

The *psignifit4* Matlab package was used to fit psychometric curves for each participant for all three conditions (<https://github.com/wichmann-lab/psignifit/>) [22]. A logistic fit was obtained based on the proportion of trials that were perceived as stiffer for each test spring [16], [23]. Discrimination thresholds or Just Noticeable Differences (JNDs) were obtained using the stimulus value corresponding to the 75% proportion correct point on the logistic curve [16] using the following relationship:

$$JND(\%) = (\Delta I/I) \cdot 100 \quad (2)$$

where, I represents the intensity of the reference stimuli (2 mNm/deg) and ΔI represents the difference between the reference stimuli and the stimulus value corresponding to the 75% proportion correct response rate.

F. Motion Analysis

A motion analysis was performed, to understand potential confounding effects of kinematic differences in active exploration on the stiffness perception thresholds of the two hands. The mean peak angular displacement was measured for each participant's left hand (s_l) and right hand (s_r).

G. Statistical Analysis

Statistical analyses were performed in MATLAB R2018b and IBM SPSS Statistics 26. For each variable, outliers were defined as values that were greater than two standard deviations away from the group median for each condition and were removed from the analyses. Assumptions of normality were tested using the Shapiro Wilk test, when required. The following statistical tests were performed:

- 1) A paired t-test was performed to compare the Left and Right hand stiffness discrimination JNDs.
- 2) The Wilcoxon's signed rank test was used to look for any significant differences in the mean peak displacements for the Left hand (s_l) and the Right hand (s_r).

III. RESULTS

We found one outlier in our experiment based on the mean peak angular displacement in the left hand condition (s_l). This participant pronated their left hand only to a mean peak displacement of 45.82 degrees across all trials. The springs in this study were designed with an expected pronation of approximately 70-90 degrees, consonant with our prior investigation on unconstrained exploration [14], and consistent with the rest of the participants in this study. Since the torques displayed for this participant were too low to accurately measure perceptual thresholds for our selected parameters, they were excluded from any analysis.

For the remaining N=13 participants, no one in our study failed a catch trial more than once and hence, no participants were excluded from the experiment on the basis of this requirement. All participants displayed a laterality index greater than 30, indicating right hand dominance. One participant was identified as an outlier based on the Left hand JND values. Data from this participant was excluded from any statistical analyses, resulting in a final sample size of n=12. No additional outliers were identified for either exploration conditions. Results for the JNDs and Mean Peak Angular Displacement of each hand for both conditions, along with the Laterality Index for all participants are presented in Table I.

A. Psychophysical Performance

The assumption of normality was met based on the results of the Shapiro Wilk test for JND values for both left and right hand exploration conditions ($p > 0.05$). A two-tailed paired t-test revealed a statistically significant difference in the Left and Right JNDs, where the Right hand JNDs were higher than the Left hand JNDs with a mean difference of 1.73%, 95% CI [0.19,3.26], $t(11)=2.47$, $p=0.031$, $d=0.70$ (see Fig. 4).

B. Motion Analysis

The data for mean peak displacement for the Left hand (s_l) failed to meet the assumptions of normality, so non-parametric tests were used to compare peak displacements between the two hands of each participant. The Wilcoxon signed rank test revealed no statistically significant differences in the mean peak angular displacement of the Left hand and the Right hand exploration conditions ($p > 0.05$).

IV. DISCUSSION

In this study, we investigated the perceptual asymmetries for stiffness cues in active unimanual exploration of torsion springs. While unimanual perception has been studied extensively for tasks involving proprioceptive and cutaneous stimuli, the relative sensitivity of each exploration strategy

TABLE I
INDIVIDUAL RESULTS FOR JND, MEAN PEAK ANGULAR
DISPLACEMENT,
AND LI VALUES FOR ALL PARTICIPANTS

Participant	JND _l (%)	JND _r (%)	s_l^* (deg)	s_r^* (deg)	LI
P1	15.23	14.47	80.13 (9.86)	82.44 (7.21)	100
P2	12.13	14.00	74.20 (4.82)	62.57 (3.85)	92
P3	8.63	13.03	70.21 (9.25)	86.22 (7.37)	92
P4	7.11	9.32	77.48 (4.52)	83.42 (4.70)	83
P5	7.52	9.90	75.05 (6.21)	68.41 (4.83)	80
P6	7.52	7.46	107.99 (6.77)	83.85 (5.64)	36
P7	8.70	9.77	101.59 (4.61)	100.01 (4.75)	100
P8	5.62	3.90	84.94 (7.73)	84.12 (8.83)	81
P9	5.89	12.65	84.52 (5.40)	67.67 (2.78)	67
P10	2.37	6.39	82.47 (5.31)	86.86 (3.38)	90
P11	5.42	5.93	74.38 (7.98)	69.40 (5.30)	80
P12**	17.42	11.69	84.74 (4.78)	74.07 (3.48)	100
P13	5.81	5.85	101.54 (9.79)	98.74 (6.98)	89
Mean	7.66	9.39	84.54	81.14	72.84
[STD]	± 3.21	± 3.40	± 11.93	± 11.45	± 40.09

*parenthesis include standard deviation in angular displacement for each participant.

**denotes outlier participant, data from this participant is not included in group mean and standard deviation.

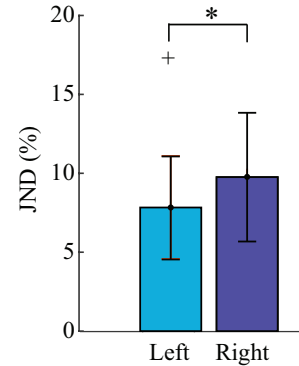


Fig. 4. Mean JND (%) for the two exploration conditions. Error bars represent one standard deviation. Outliers are represented using the "+" symbol.

for kinesthetic cues is not well understood. Our findings highlight a key difference in the formation of stiffness percepts for both hands that closely parallels prior research that found similar differences between the hands in the formation of curvature percepts [1]. In the remainder of this discussion, we will focus on comparing our methods and results to prior studies in the haptics and motor performance literature.

A. Psychophysical Methods

We used an asymmetric variant of the method of constant stimuli, where test springs were always stiffer than the reference spring. Asymmetric designs offer the advantage of obtaining accurate curves with smaller number of trials. We made this choice as a means of carefully balancing the trade-off between accurate psychophysical estimates and participants' mental and physical fatigue. In addition, considering the stiffness of our reference spring, evaluating test springs lower than our reference spring would have resulted in test

springs very close to the detection threshold for stiffness, which would have adversely affected the accuracy of our findings. Since Weber's law has been proven to be applicable for stiffness perception, it can be assumed that the upper half and lower half psychometric curves for stiffness perception are symmetric. The point of subjective equality (PSE) can be assumed to be at $p_s=0.5$ since both the test and reference springs are produced on the same setup. By following a "stiffer than" approach instead of "same-different," along with randomization of the presentation order, we reduced the likelihood of bias resulting from test springs that are always greater than the reference. This approach is therefore similar, regarding bias, to single adaptive staircases widely used in psychophysical experiments, where the test stimulus is either greater (descending staircase) or smaller (ascending staircase) than the reference in most trials [24]–[26]. The choice of an asymmetric design is consistent with the psychophysical literature on threshold estimation [27]–[29]. The introduction of catch trials is also an accepted method to mitigate potential confounds from a response bias.

B. Perceptual Asymmetry

We found significant differences in stiffness perception thresholds for active unimanual exploration. To the best of the authors' knowledge, this is the first time that a significant perceptual asymmetry has been reported between left and right hand for stiffness perception. In particular, we found that participants in our study had significantly smaller thresholds when discriminating between torsion springs using their left hands compared to their right hand. The notion of left hand perceptual dominance is consistent with prior studies on proprioceptive cues like curvature discrimination and reaching tasks [1], [10]. While our results confirm that a form of perceptual-handedness occurs for kinesthetic cues, a subsequent investigation will be required to fully understand how motor dominance might relate to perceptual dominance, since disparities between left-handed and right-handed participants have previously been reported for haptic tasks [12], [13], [30]. Leib et al. have previously shown that perceptual biases induced by haptic feedback delays depend on the hand that was used for exploration but not on hand dominance [31]. As such, our experiment was not designed to study the potential effects of motor dominance. The left hand perceptual advantage observed in our experiment, could there be a result of factors other than the right hand motor dominance of our participant pool.

These findings have direct implications on how we study human perception, in particular when contralateral limb-matching procedures are utilized for estimating perceptual thresholds. In these experimental procedures, participants are asked to tune haptic cues displayed on one limb to match the cues perceived on the other limb [5], [32]. Furthermore, these procedures are often designed around guidelines that require the matching and standard stimuli to be presented simultaneously, and require the matching variable to be continuous [15]. Our results raise important questions on whether these guidelines alone are sufficient for psychophysical evaluations

of this kind. Our findings may also aid in the design of haptic feedback devices. With the rise of haptic feedback in teleoperated environments and clinical therapies, targeting the hand with a perceptual advantage could potentially allow for improved user experience and performance.

C. Motion Analysis

Our experiment was designed for free active exploration of torsion springs with minimal motion constraints to mimic real life exploration. Since there were no significant differences in the displacement for the Left and Right explorations conditions, the potential impact of terminal force cues [17] is minimized, adding further validation to our finding of perceptual differences between the two hands. In addition, we know from our prior work that similar minimally constrained active exploration in humans can result in consistent motion patterns, and that differences in exploration velocity do not have a significant effect on perception [14]. This is also consistent with the human tendency to move our effectors at a consistent frequency [33]. Given the fact that we observed no notable differences in exploration strategies between the two hands, we believe that the differences observed in perception in our experiment can not be explained by any motion confounds that may have been introduced by our choice of minimally constrained active exploration.

V. FUTURE WORK

While this study helps us understand how percepts are formed from the two hands, it also opens up the need to investigate how motor dominance might contribute to percept formation during active exploration. Exploration of stiffness cues offer a distinct advantage as it combines a motor dominant task (displacement) with perception of a predominantly kinesthetic cue (force). Future studies should therefore investigate the potential impact of hand dominance on perceptual asymmetries of kinesthetic cues under active exploration. There is also the need to study how these perceptual asymmetries may be accounted for when used in haptic feedback devices, and any effects they may have on task performance of the human in the loop in these systems. We also believe that the observed perceptual asymmetries raise questions on how such differences may be resolved in bimanual active exploration of similar cues. There is a lack of consensus on how we form precepts when using bimanual exploration strategies. Whether we use the process of sensory selection to favor the perceptually dominant hand or the process of sensory integration to optimally combine sensory data from both hands [18], [34]. For stiffness perception, the study of bimanual percept formation will present an added layer of complexity since the stiffness precepts are informed by both the proprioceptive sense of position and the kinesthetic sense of force.

VI. CONCLUSION

In summary, we have found evidence of perceptual asymmetries in our upper limbs when exploring stiffness cues using unimanual active exploration. These findings can

serve as important considerations for experimental design involving psychophysical methods like limb-matching, where participants are required to match the stimuli received on one hand to the reference stimuli presented on the other hand. Often used to obtain perceptual thresholds, this method can be prone to bias if similar sensations are perceived differently by both hands. These findings can also prove useful for haptic designers who desire to provide haptic feedback that is consonant with the manner in which the user may perceive it. As increasingly complex haptic feedback mechanisms are deployed in critical fields like surgery, prosthetics and defense, it is essential to account for the differences in perceptual thresholds between the two hands, and any potential effects that it may have on task performance in these settings. We believe that our findings regarding kinesthetic perception complete the theory of perceptual asymmetry across all the primary haptic perception domains, previously proven only cutaneous and proprioceptive cues.

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